

CONTEMPORARY BUSINESS MATHEMATICS with Canadian Applications

S. A. Hummelbrunner Kelly Halliday Ali R. Hassanlou




TWELFTH EDITION

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MATHEMATICS WITH
CANADIAN APPLICATIONS**

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CONTEMPORARY BUSINESS MATHEMATICS WITH CANADIAN APPLICATIONS

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12th Edition

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PREFACE

INTRODUCTION

Contemporary Business Mathematics with Canadian Applications is intended for use in introductory mathematics of finance courses in post-secondary business management, marketing, accounting, and finance programs. It also provides a review of basic mathematics.

The primary objective of the text is to increase the student's knowledge and skill in the solution of practical financial and operational problems encountered in operating a business.

ORGANIZATION

Contemporary Business Mathematics with Canadian Applications is a teaching text using problem-identification and problem-solving approaches. The systematic and sequential development of the material is illustrated by examples that show a step-by-step approach to solving the problem. The detailed solutions are presented in a visually clear and colourful layout that allows learners to monitor their own progress in the classroom or in independent study.

Each topic in each chapter is followed by practice exercises containing numerous drill questions and application problems. At the end of each chapter, Review Exercises, Self-Test, Challenge Problems, and a Case Study integrate the material presented.

The first four chapters and Appendix I (Further Review of Basic Algebra) are intended for students with little or no background in algebra and provide an opportunity to review arithmetic and algebraic processes.

The text is based on Canadian practice, and reflects current trends using available technology—specifically the availability of preprogrammed financial calculators. Students using this book should have access to calculators having a power function and a natural logarithm function. The use of such calculators eliminates the constraints associated with manually calculating results using formulas.

In solving problems involving multiple steps, often values are determined that will be used in further computations. Such values should not be rounded and all available digits should be retained in the calculator. Using the memory functions of the calculator enables the student to retain such non-rounded values.

When using the memory, the student needs to be aware that the number of digits retained in the registers of the calculator is greater than the number of digits displayed. Depending on whether the memory or the displayed digits are used, slight differences may occur.

Students are encouraged to use preprogrammed financial calculators. The use of these preprogrammed calculators facilitates the solving of most financial problems and is demonstrated extensively in Chapters 9 to 16.

F. Calculating the numerical value of the compounding factor $(1 + i)^n$

The numerical value of the compounding factor, $(1 + i)^n$, can now be computed using an electronic calculator. For calculators equipped with the exponential function feature, $e^{x\ y}$, the numerical value of the compounding factor can be computed directly.

STEP 1: Enter the numerical value of $(1 + i)$ in the keyboard. Tip: enter i and then add 1.

STEP 2: Press the exponential function key $e^{x\ y}$.

STEP 3: Enter the numerical value of n in the keyboard.

STEP 4: Press $=$.

STEP 5: Read the answer in the display. Continue calculating or save as needed.

The numerical values of the compounding factors in Example 9.1C are obtained as follows:

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
STEP 1: Enter	$1 + 0.05$	$1 + 0.07/2$	$1 + 0.12/4$	$1 + 0.105/12$	$1 + 0.08/4$	$1 + 0.09/2$	$1 + 0.084/365$
STEP 2: Press	$e^{x\ y}$	$e^{x\ y}$	$e^{x\ y}$	$e^{x\ y}$	$e^{x\ y}$	$e^{x\ y}$	$e^{x\ y}$
STEP 3: Enter	14	30	50	125	18	7	730
STEP 4: Press	$=$	$=$	$=$	$=$	$=$	$=$	$=$
STEP 5: Read	1.81992	2.00794	4.86596	3.79447	1.21094	1.38048	1.22786

NEW TO THIS EDITION

The Twelfth Edition of Hummelbrunner/Halliday/Hassanlou, *Contemporary Business Mathematics with Canadian Applications*, includes updates based on changes in current practices in Canadian finance and business and the needs of students and instructors using this book.

- This edition continues to clarify the consistent approach to rounding rules. The **Student's Reference Guide to Rounding and Special Notations** (pages xix–xxii) gives a clear explanation of the rounding conventions used throughout the text. Additional *Pointers and Pitfalls* boxes are placed in key areas to remind students about the rounding conventions and exceptions in practice.
- The text and solutions manual have been thoroughly technically checked for accuracy and consistency with the rounding approach.
- Tables, charts, and further diagrams have been added to enable the learner to visualize the problems and the solutions.
- Numerous new examples and exercises have been added.
- To help students better understand and solve systems of linear equations, the order of presenting concepts has been changed in Chapter 4. We first introduce an easier concept of graphing linear solutions and continue with algebraic solutions of systems of linear equations with two unknown variables. Finally, solving systems of linear inequalities is presented.
- The cost data in the main example for explaining break-even analysis in Chapter 5 have been revised to eliminate confusion and help students understand various costs of doing business, perform cost-volume-profit analysis, and calculate the break-even point in a business.
- Canadian references have been emphasized in *Business Math News Boxes* and website references.
- Interest rates reflect current investment and borrowing rates.

Many examples and exercises have been updated, rewritten, and expanded. To enhance the building-block approach, exercises are ordered to link the topics and the solved examples. Help references have been expanded to link selected exercises to solved examples.

Specifically, in **Chapter 1** (Review of Arithmetic), prices, salaries, and wages have been updated. Revised rates and calculations for GST/PST/HST have been included to incorporate new legislation for 2018 and property tax terminology and valuations have been updated. Weighted average examples and exercises have been expanded and additional drill questions have been added throughout Chapter 1. The Business Math News Box on National Salary Comparisons has been updated and moved here from Chapter 3.

In **Chapter 2** (Review of Basic Algebra), the chapter-opening vignette emphasizes why business students need algebra, algebra explanations have been expanded, and new diagrams and *Pointers and Pitfalls* clarify these fundamental approaches. Language and math scaffolding strategies have been emphasized in this chapter to improve students' understanding of key concepts and relationships. Calculator solutions have been introduced for several examples and formulas have been simplified.

In **Chapter 3** (Ratio, Proportion, and Percent), a consistent approach for calculating proportions has been introduced, and the percentage-base-rate triangle has been included as a useful aid for determining percentages. Sections in the chapter have been re-ordered to improve student understanding. For example, instructions for calculating the base appear before calculating the percentage rate. Currency conversion rates, prices, CPI numbers, and personal income taxes have been updated. A new Business Math News Box highlights the variability of apartment rental rates in major Canadian cities.

In **Chapter 4** (Linear Systems), the order of the first three subsections has been changed to first introduce graphing linear equations and then explain algebraic solutions

to linear systems in two variables. The substitution and elimination methods for solving the point of intersection of two linear systems has been emphasized.

The order of **Chapter 5** and **Chapter 6**, which was switched in the Tenth Edition, is retained to improve the flow of content from Linear Systems (Chapter 4) to Cost-Volume-Profit Analysis and Break-Even (Chapter 5). This chapter has been changed significantly with a comprehensive example that starts at the beginning and continues throughout the chapter. In Chapter 5, a new *Business Math News Box* ties the concepts learned in this chapter to the legalization of marijuana in Canada. The Chapter 5 opening vignette appropriately includes an example that connects with the new *Business Math News Box* presented later in the chapter. The number of formulas has been reduced, and solutions to examples use a simplified approach for calculating break-even.

Chapter 6 (Trade Discounts, Cash Discounts, Markup, and Markdown) explanations and diagrams have been revised to clarify key concepts. *Pointers and Pitfalls* boxes provide tools to help students rearrange formulas, determine the number of days in a discount period, and calculate markup. A sample invoice demonstrates payment terms and cash discounts. EOM and ROG examples, are retained to help students understand the terminology and concepts as they are being used in practice by businesses. Beginning in this chapter, instructions in the text that previously asked students to “find” specific variables have been replaced by more mathematical language such as “solve,” “calculate,” and “determine.”

In **Chapter 7** (Simple Interest), a new Business Math News Box outlining the perils of “buy now, pay later” plans has replaced an outdated box based on the Canada Savings Bond Program. Dates and interest rates have been updated and new exercises have been added, with exercises referenced to examples. Additional tools for choosing focal dates and calculating number of days have been added.

In **Chapter 8** (Simple Interest Applications), a new opening vignette leaves way for a Business Math News Box focused on calculating the annual percentage rate of charge for payday loans. Comments on credit ratings, credit scores, home equity lines of credit, and new exercises calculating unpaid balances have been added. Treasury bill interest rates have been updated to reflect current rates. A new subsection on commercial paper has been included in this edition.

In **Chapter 9** (Compound Interest—Future Value and Present Value), visual explanations for drawing timelines and selecting focal dates have been expanded. The introduction to Future Value, and explanation of the periodic rate of interest, have been simplified. The relationship between n and m has been clarified. Additional examples and questions using weekly and bi-weekly compounding periods have been included, along with questions featuring changing interest rates. A new Business Math News Box using data for fixed- and variable-rate Guaranteed Investment Certificates (GICs) provides examples of escalating interest rates.

In **Chapter 10** (Compound Interest—Further Topics), formula rearrangement is emphasized. A Pointers and Pitfalls box has been added to explain how to calculate the “true cost” of borrowing. Drill questions from previous editions for calculating effective and equivalent interest rates have been added back to the section exercises.

In **Chapter 11** (Ordinary Simple Annuities) and in **Chapter 12** (Ordinary General Annuities), new Pointers and Pitfalls boxes have been added to help students set up equations at the focal date. Advanced questions have been added to the chapter exercises, including changes in interest rates and changes in payment size, as well as calculating the size of the final payment in an annuity. The updated Business Math News Boxes feature credit card minimum payment myths and whether or not it is better to lease or buy a car.

In **Chapter 13** (Annuities Due, Deferred Annuities, and Perpetuities), explanations, diagrams, and calculations for annuities due are simplified. New examples with diagrams have been added, including reference to investments in preferred shares. New Pointers and Pitfalls boxes remind the reader how to calculate the number of payments in an

annuity, and how to calculate present value of perpetuities when the size of the payment increases or decreases at a constant rate.

In **Chapter 14** (Amortization of Loans, Including Residential Mortgages), a new section with a diagram develops the skills to calculate the interest, principal, and balance for a period, and bridges between calculating the payment and constructing the amortization schedule. The introduction to residential mortgages has been updated to reflect current legislation on mortgage insurance and stress testing. The “sinking funds” concept covered in this chapter of the Eleventh Edition is now removed from this chapter and included in Chapter 15. Examples and exercises have been reordered and clarified to enhance building-block learning.

In **Chapter 15** (Bond Valuation and Sinking Funds), explanation of basic concepts has been expanded to answer the “why?” and “how?” questions. The order has been changed, with a focus on calculation of bond price under different conditions. An introductory section has been added for concept comprehension. Calculating the purchase price of a bond has been separated into two sections based on whether or not the market rate equals or does not equal the bond rate. Instructions and examples have been provided on how to use the Texas Instrument BA II PLUS calculator to calculate the purchase price of a bond on an interest payment date or between interest payment dates. The “sinking funds” concept that was introduced in Chapter 14 in the Eleventh Edition is now included in this chapter to help students understand how corporations plan to have funds available to pay back their issued bonds on the maturity date.

In **Chapter 16** (Investment Decision Applications), explanations begin the section on Net Present Value, followed by introductory, then more advanced, applications. Repetitive calculator instructions have been eliminated. Computing the Rate of Return by manual methods has been condensed and new visuals have been added. Instructions for using the cash flow analysis of the Texas Instrument BA II PLUS financial calculator and Excel’s NPV and IRR functions have been included, with an example for each function to reinforce learning.

COMPREHENSIVE CASE STUDIES

Comprehensive case studies for each part of the book have been created. The questions within each case study have been separated by chapter or group of chapters to facilitate the use of these case studies by those institutions that include only some of the topics in their course syllabus. With the questions separated and identified by chapter, these institutions can use part of the case study in their courses.

Part 1 Mathematics Fundamentals and Business Applications

Til Debt Do Us Part host Gail Vaz-Oxlade has made it her mission to help couples who are headed for disaster get out of debt. Questions for each of Chapters 1–4 are included.

Part 2 Mathematics of Business and Management

A sporting equipment manufacturer and retailer, SportZ Ltd., is based in Alberta. Questions for each of Chapters 5–8 are included.

Part 3 Mathematics of Finance and Investment

Based in Ontario, Lux Resources Group, Inc., rents and sells construction equipment. Questions for each of Chapters 9–16 are included.

In general, interest rates used reflect the current economic climate in Canada. Calculator tips and solutions have been updated or clarified. Spreadsheet instructions and Internet website references have been updated. *Pitfalls and Pointers* have been included to assist in performing tasks and interpreting word problems, and sections have been rewritten to clarify the explanations. Many more word problems have been added and references to solved examples increased. *Business Math News Boxes* and Case Studies

have been updated. Examples involving both business and personal situations are included. The pedagogical elements of the previous edition have been retained. In response to requests and suggestions by users of the book, a number of features for this edition have been included. They are described next.

FEATURES

UPDATED!

- A new colourful and student-friendly design has been created for the book, making it more accessible and less intimidating to learners at all levels.

UPDATED!

- Any preprogrammed financial calculator may be used, but this edition includes extensive instructions for using the Texas Instruments BA II PLUS financial calculator. Equivalent instructions are given in Appendix II for the Sharp EL-738C and the Hewlett-Packard 10BII+ financial calculators.
- Each part opens with an introduction to the upcoming chapters and a discussion of the rounding conventions that are relevant to these chapters.
- A set of Learning Objectives is listed at the beginning of each chapter. The corresponding Learning Objectives are also indicated for each Review Exercise, allowing students to see which aspects of the chapter they have mastered.
- Each chapter opens with a description of a situation familiar to students to emphasize the practical applications of the material to follow.
- A **Business Math News Box** is presented in every chapter. This element consists of short excerpts based on material appearing in newspapers, magazines, or websites, followed by a set of questions. These boxes demonstrate how widespread business math applications are in the real world.
- The **Pointers and Pitfalls** boxes emphasize good practices, highlight ways to avoid common errors, show how to use a financial calculator efficiently, or give hints for tackling business math situations to reduce math anxiety.
- Numerous **Examples with worked-out Solutions** are provided throughout the book, offering easy-to-follow, step-by-step instructions.
- **Programmed solutions** using the Texas Instruments BA II PLUS calculator are offered for most examples in Chapters 9 to 16. Since this calculator display can be pre-set, it is suggested that the learner set the display to show six decimal places to match the mathematical calculations in the body of the text. Both mathematical and calculator solutions

by depressing the key representing the unknown variable, depending on the calculator model.

Table 8.3 Financial Calculator Function Keys That Correspond to Variables Used in Compound Interest Calculations (© C. A. Henderson)

Variable	Algebraic Symbol	Function Key			
		TI BA II PLUS	Sharp EL-738C	HP 10BII+	
The number of compounding periods	n	N	N	N	
The rate of interest ¹	i	I/Y C/Y	I/Y	I/Y%	
The periodic security payment ²	PMT	PMT	PMT	PMT	
The present value or principal	PV	PV	PV	PV	

CHAPTER 10
Compound Interest—Further Topics

LEARNING OBJECTIVES

Upon completing this chapter, you will be able to do the following:

- Determine the number of conversion periods and solve equivalent rates.
- Compare periodic and nominal rates of interest.
- Compute effective and equivalent rates of interest.

All we want is a loan and is wondering how much this loan is costing him in interest. If the bank is paid back over a long period of time, how much interest will he have to pay? Can he reduce the amount of interest? Most Canadians hold debt and, with currently low interest rates, pay relatively low amounts of interest. If interest rates were to rise, how much more interest would he have to pay? If he made the same payment, how much longer would it take to repay his debt? How much debt is he really? Many people reduce their debt by making earlier or extra payments. The key to managing debt is knowledge of how much is owed, what interest rate is charged, and how long it takes to repay the debt.

Getting out of debt, or at least getting your finances back on track, has many long-term benefits. These include lower monthly payments, reduced interest rates and fees, getting out of debt faster, and improving your credit rating.

BUSINESS MATH NEWS lululemon athletica inc. Announces Third Quarter Fiscal 2018 Results

lululemon athletica, a high-end retail chain dedicated to yoga and fitness apparel, recently experienced several years of record growth.

Founded and headquartered in Vancouver, British Columbia, lululemon manufactures and sells technical athletic wear aimed primarily at active men and women who are willing to pay premium prices for workout gear. These high prices have allowed the retailer to maintain a gross margin annually above 50% since 2003.

Although the initial goal was to have only one store, consumer demand has resulted in multiple locations. Most stores are located in major cities in Canada, the United States, Australia, New Zealand, and the United Kingdom, as well as in Singapore. lululemon's huge sales growth can be largely attributed to the company's extensive store expansion.

To reach even more customers, lululemon launched a successful e-commerce operation on its company website in 2009. After posting its best-ever first quarter, lululemon said that its practices were to grow existing stores and to invest more in its thriving online division. On its company website, lululemon reported a 30% increase in traffic to its e-commerce site in 2018 with its e-commerce efforts netting US \$476.9 million in sales, compared to US \$421.1 million the year before.

The company offers regular-priced in-store products online. It also offers discounted men's and women's athletic wear under its "we made too much" clearance link.

The following items were recently discounted on the lululemon website:

- Women's Wunder Under Pant: \$69.00 CAD (was \$88.00)
- Women's City Striped Jacket: \$89.00 CAD (was \$118.00)
- Men's Cardio SS Tech Top: \$24.00 CAD (was \$58.00)
- Men's Performance Jacket: \$44.00 CAD (was \$88.00)

QUESTIONS

1. lululemon athletica inc. reported that its net revenue for the third quarter of 2018 was \$742.7 million, an increase of 21% compared to the third quarter of fiscal 2017. Calculate the net revenue for the third quarter of 2017 (rounded to the nearest hundred thousand dollars).
2. Calculate e-commerce revenue as a percentage of total revenue for the period.
3. Calculate the rate of discount for each of the four clearance items listed under the "we made too much" link (rounded to 2 decimal places).
4. Assuming lululemon's overhead is 30% of the regular selling price, and that the cost of the Women's Wunder Under Pant is \$36, determine:
 - (a) the markup, the overhead, and the profit for this item sold at the regular selling price.

POINTERS & PITFALLS

You can use the **BREAK-EVEN** function of a Texas Instruments BA II PLUS financial calculator to determine the break-even point. In this function, FC refers to the total fixed cost for the period, VC refers to the unit variable cost, P refers to the unit price, PPT is the resulting profit, and Q is used to input or calculate the quantity or number of units. For example, consider Example 5.1D using a financial calculator, here:

$FC = 8640.00$ **Enter** **↓**
 $VC = 50.00$ **Enter** **↓**
 $P = 50.00$ **Enter** **↓**
 $PPT = 0$ **Enter** **↓**
 CPT **Q** = 432 units

Any four of the five variables may be entered. You can then compute a value for the fifth variable.

EXAMPLE 13.2A What is the accumulated value after five years of payments of \$20 000 made at the beginning of each year if interest is 7% compounded quarterly?

SOLUTION


$PMT = 20\,000.00$, $n = 5$, $i = 4$, $P/Y = 1$, $C/Y = 4$, $I/Y = 7$
 $j = \frac{7\%}{4} = 1.75\% = 0.0175$
 The equivalent annual rate of interest
 $p = 1.0175^4 - 1 = 1.071859 - 1 = 0.071859 = 7.1859\%$
 $FV(\text{due}) = 20\,000.00 \left(\frac{1.071859^5 - 1}{0.071859} \right) (1.071859)$ — substituting in Formula 13.3
 $= 20\,000.00(5.772109)(1.071859)$
 $= 115\,442.1869(1.071859)$
 $= \$123\,737.75$

PROGRAMMED SOLUTION (Set P/Y = 1, C/Y = 4) **↓** **FV** **↓**
 ("BGN" mode) **Set P/Y = 1, C/Y = 4** **↓** **20000** **↓** **5** **↓** **7** **↓** **CPT** **FV** **↓** **123737.7508**

The accumulated value after five years is \$123 737.75.

UPDATED!

for all Exercises, Review Exercises, and Self-Tests are included in the Instructor's Solutions Manual. An icon highlights information on the use of the BA II PLUS calculator.

- **Key Terms** are introduced in the text in boldface type. A Glossary at the end of each chapter lists each term with its definition and a page reference to where the term was first defined in the chapter.
- Main Equations are highlighted in the chapters and repeated in a Summary of Formulas at the ends of the chapters.
- A list of the Main Formulas can be found on the study card bound into this text.
- An **Exercise set** is provided at the end of each section in every chapter. In addition, each chapter contains a **Review Exercise set** and a **Self-Test**. Answers to all the odd-numbered Exercises, Review Exercises, and Self-Tests are given at the back of the book.
- Also included in this edition are references to solved Examples in several chapters, which are provided at the end of key exercises. Students are directed to specific examples so they can check their work and review fundamental problem types.
- Exercises and Review Exercises that are marked with a  are also available on MyLab Math. Students have endless opportunities to practise many of these questions with new data and values every time they use MyLab Math.
- A set of **Challenge Problems** is provided in each chapter. These problems give users the opportunity to apply the skills learned in the chapter to questions that are pitched at a higher level than the Exercises.
- Sixteen **Case Studies** are included in the book, at the end of each chapter. They present comprehensive realistic scenarios followed by a set of questions and illustrate some of the important types of practical applications of the chapter material. Sixteen additional case studies can be found on MyLab Math.

11.1 INTRODUCTION TO ANNUITIES

A. Basic concepts

In this textbook, an **annuity** is a series of equal payments, made at periodic intervals. The length of time between the successive payments is called the **payment interval** or **payment period**. The length of time from the beginning of the first payment interval to the end of the last payment interval is called the **term of an annuity**. The amount of each of the regular payments is called the **periodic payment**, or **periodic rent**.

When performing annuity calculations, the timing of payments must be considered. Depending on the frequency and regularity of payments, different formulas will be used in annuity calculations. When a payment is made only once, it is treated as either the present value, PV, or the future value, FV, of a calculation. When there are a series of payments, it must be determined if the payments are equal amounts and are paid at the same time within each payment interval of the term. If the payment is equal and periodic, it is treated as the periodic payment, PMT, of an annuity calculation. The types of annuities are described in Section B below.

GLOSSARY

Accumulated value of one dollar per period or Accumulation factor for annuities the expression $\frac{(1+i)^n - 1}{i}$ (p. 422)

Annuity a series of payments, usually equal in size, made at equal periodic time intervals (p. 418)

Annuity certain an annuity for which the term is fixed (p. 419)

Discounted value of one dollar per period or Discount factor Down payment the portion of the purchase price that is supplied by the purchaser as an initial payment (p. 415)

General annuity an annuity in which the conversion (or compounding) period is different from the payment interval (p. 419)

Ordinary annuity an annuity in which the payments are made at the end of each payment interval (p. 418)

Payment interval the length of time between successive payments (p. 418)

$$\begin{aligned} \text{CONTRIBUTION MARGIN} &= \text{SELLING PRICE} - \text{VARIABLE COST} \\ \text{PER UNIT} & \quad \text{PER UNIT} \quad \text{PER UNIT} \\ \text{OR,} & \quad \text{CM PER UNIT} = \text{SP} - \text{VC} \end{aligned} \quad \text{Formula 5.2}$$

When the contribution margin per unit is multiplied by the number of units, the result is the **total contribution margin**.

$$\begin{aligned} \text{TOTAL CONTRIBUTION} &= \left(\frac{\text{SELLING PRICE}}{\text{PER UNIT}} - \frac{\text{VARIABLE COST}}{\text{PER UNIT}} \right) \times \text{VOLUME} \\ \text{MARGIN} & \quad \text{OR, TOTAL CM} = (\text{SP} - \text{VC}) \times X \end{aligned} \quad \text{Formula 5.3}$$


















Using the contribution margin format, Formula 5.1 can be rewritten as

$$(\text{SP} - \text{VC}) \times X - \text{FC} = \text{PFT} \quad \text{Formula 5.1B}$$

MyLab Math

Use MyLab Math to practise any of the chapter exercises marked with  as often as you want. The guided solutions help you calculate an answer step by step. You'll find a personalized study plan and additional interactive resources to help you master Business Math.

REVIEW EXERCISE

1.  At what nominal rate of interest compounded monthly will \$400 earn \$100 interest over four years?
2.  At what nominal rate of interest compounded quarterly will \$300 earn \$80 interest in six years?
3.  Determine the equated date at which payments of \$500 due six months ago and \$600 due today could be settled by a payment of \$1300, if interest is 9% compounded monthly.
4.  Determine the equated date at which two payments of \$600 due four months ago and \$400 due today could be settled by a payment of \$1100, if interest is 7.25% compounded semi-annually.
5.  In what period of time will money triple at 10% compounded semi-annually?
6.  In how many months will money double at 8% compounded monthly?
7.  What nominal rate of interest compounded monthly is equivalent to an effective rate of 6.26?
8.  What nominal rate of interest compounded quarterly is equivalent to an effective rate of 3.996?
9.  Calculate the nominal annual rate of interest:
 - (a) at which \$2500 will grow to \$4000 in eight years compounded quarterly;
 - (b) at which money will double in five years if compounded semi-annually;
 - (c) if the effective annual rate of interest is 9.2% and compounding is done monthly;
 - (d) that is equivalent to 8% compounded quarterly.
10.  Calculate the nominal annual rate of interest:
 - (a) at which \$1500 will grow to \$1800 in four years compounded monthly;
 - (b) at which money will double in seven years if compounded quarterly.
- (c) if the effective annual rate of interest is 7.75% and compounding is done monthly;
- (d) that is equivalent to 6% compounded quarterly.
11.  Compute the effective annual rate of interest:
 - (a) for 4.5% compounded monthly;
 - (b) at which \$2000 will grow to \$3000 in seven years if compounded quarterly.
12.  Compute the effective annual rate of interest:
 - (a) for 6% compounded annually;
 - (b) at which \$1100 will grow to \$2000 in seven years if compounded monthly.
13.  What is the nominal annual rate of interest compounded monthly that is equivalent to 8.5% compounded quarterly?
14.  What is the nominal annual rate of interest compounded quarterly that is equivalent to an effective annual rate of 5%?
15.  Patrick has \$2000 to invest. Which of the following options should he choose?
 - (a) 4% compounded annually
 - (b) 3.75% compounded semi-annually
 - (c) 3.5% compounded quarterly
 - (d) 3.25% compounded monthly
16.  (a) How many years will it take for \$7500 to accumulate to \$9517.39 at 3% compounded semi-annually?
 - (b) Over what period of time will money triple at 9% compounded quarterly?
 - (c) How many years will it take for a loan of \$10 000 to amount to \$13 684 at 10.5% compounded monthly?
17.  Matt agreed to make two payments—a payment of \$2000 due in nine months and a payment of \$1500 in a year. If Matt makes a payment of \$1800 now, when should he make a second payment of \$1700 if money is worth 8% compounded quarterly?

EXERCISE 9.2

MyLab Math

1. What is the maturity value of a five-year term deposit of \$5000 at 3.5% compounded semi-annually? How much interest did the deposit earn?
2. How much will a registered retirement savings deposit of \$1500 be worth in 15 years at 3.48% compounded quarterly? How much of the amount is interest?
3. Reece's parents made a trust deposit of \$500 on October 31, 2004, to be withdrawn on Reece's twenty-first birthday on July 31, 2025. What will be the value of the deposit on that date at 7% compounded quarterly?

CHALLENGE PROBLEMS

1. Two consecutive price reductions of the same percent reduced the price of an item from \$25 to \$16. By what percent was the price reduced each time?
2. Luis ordered four pairs of black socks and some additional pairs of blue socks from a clothing catalogue. The price of the black socks per pair was twice that of the blue. When the order was filled, it was found that the number of pairs of the two colours had been interchanged. This increased the bill by 50% (before taxes and delivery charges). Determine the ratio of the number of pairs of black socks to the number of pairs of blue socks in Luis' original order.
3. Following a 10% decrease in her annual salary, what percent increase would an employee need to receive in future to get back to her original salary level?

TECHNOLOGY RESOURCES

MyLab Math

MyLab Math from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy-to-use format. It provides engaging experiences that personalize, stimulate, and measure learning for each student. And, it comes from an experienced partner with educational expertise and an eye on the future.

To learn more about how MyLab Math combines proven learning applications with powerful assessment, visit www.pearsonmylabandmastering.com or contact your Pearson representative.

PEARSON ETEXT

Pearson eText. The Pearson eText gives students access to their textbook anytime, anywhere. In addition to note taking, highlighting, and bookmarking, the Pearson eText offers interactive and sharing features. Instructors can share their comments or highlights, and students can add their own, creating a tight community of learners within the class.

LEARNING SOLUTIONS MANAGERS

Pearson's Learning Solutions Managers work with faculty and campus course designers to ensure that Pearson technology products, assessment tools, and online course materials are tailored to meet and anticipate specific needs. This highly qualified team is dedicated to helping schools take full advantage of a wide range of educational resources, by assisting in the integration of a variety of instructional materials and media formats. Your local Pearson Education sales representative can provide you with more details on this service program.

SUPPLEMENTS

The following instructor supplements are available for downloading from a password-protected section of Pearson Canada's online catalogue (catalogue.pearsoned.ca). Navigate to your book's catalogue page to view a list of the available supplements. See your local sales representative for details and access.

- An **Instructor's Solutions Manual** provides complete mathematical and calculator solutions to all the Exercises, Review Exercises, Self-Tests, *Business Math News Box* questions, Challenge Problems, and Case Studies in the textbook.
- An **Instructor's Resource Manual** includes Chapter Overviews, Suggested Priority of Topics, Chapter Outlines, and centralized information on all the supplements available with the text.
- **PowerPoint® Lecture Slides** present an outline of each chapter in the book, highlighting the major concepts taught. The presentation will include many of the figures and tables from the text and provides the instructor with a visually interesting summary of the entire book.
- Pearson's computerized **test banks** allow instructors to filter and select questions to create quizzes, tests or homework. Instructors can revise questions or add their own, and may be able to choose print or online options. These questions are also available in Microsoft Word format.
- A complete **Answer Key** will contain solutions for all of the exercise and self-test questions.
- **Excel Templates** will allow instructors to assign a selection of Exercises and Review Exercises to be solved using Excel spreadsheets.
- An **Image Library** will provide access to many of the figures and tables in the textbook.

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STUDENT'S REFERENCE GUIDE TO ROUNDING AND SPECIAL NOTATIONS

Developed by Jean-Paul Olivier, based on the textbook authored by Kelly Halliday and Ali Hassanlou

Universal Principle of Rounding: When performing a sequence of operations, never round any interim solution until the final answer is achieved. Only apply rounding principles to the final answer. Interim solutions should only be rounded where common practice would require rounding.

Note: Due to space limitations, the textbook only shows the first 6 decimals (rounded) of any number. Starting in Chapter 11, because the calculator display may not have sufficient space for all 6 decimals, as many decimals as possible will be shown. However, the Universal Principle of Rounding still applies.

Part 1

Section 1.2

1. For repeating decimals, use the notation of placing a period above the repeating sequence. E.g., $\frac{1}{3} = 0.333333 \dots = 0.\overline{3}$
2. For terminating decimals, if they terminate within the first 6 decimal places, then carry all the decimals in your final answer.
3. For non-terminating decimals, round to 6 decimals unless specified or logically sound to do so otherwise. If the final digits would be zeros, the zeros are generally not displayed.
4. Calculations involving money are rounded to 2 decimals as their final answer. Interim solutions may be rounded to 2 decimals if the situation dictates (for example, if you withdraw money from an account). If the calculation does not involve cents, it is optional to display the decimals.

Section 1.3

1. Calculations involving percentages will only involve 4 decimal positions since there are only 6 decimals in decimal format.

Section 1.5

1. Hourly rate calculations for salaried employees require that all the decimals should be carried until the final answer is achieved. If the solution is to express the hourly rate or overtime rate itself, then rounding to 2 decimals is appropriate.
2. Overtime hourly wage rate calculations should carry all decimals of the overtime rate until the final answer is achieved.

Section 3.7

1. Larger sums of money usually are involved in currency exchanges. Therefore, the two decimal rule for money is insufficient. To produce a more accurate result, currency exchange rates need to carry at least four decimals.
2. It needs to be recognized that not all currencies utilize the same decimals when expressing amounts.
 - (a) Final currency amounts for the Canadian Dollar, U.S. Dollar, British Pound, Euro, and Swiss Franc should be rounded to the standard two decimal places.
 - (b) Final currency amounts for the Japanese Yen should be rounded to the nearest integer, as there are no decimal amounts in their currency.

- Price per litre of gasoline is generally expressed to three decimal points ($129.9\text{¢/L} = \$1.299/\text{L}$)

Section 3.8

- As indexes are similar to percentages, an index will only have 4 decimals.

Part 2 Section 5.1

- When calculating break-even units, remember that the solution is the *minimum* number of units that must be sold. As such, any decimals must be rounded upwards to the next integer, regardless of the actual value of the decimal. For example, 38.05 units means 39 units must be sold to at least break even.

Section 7.2D

- t is always an integer. It is important to note in this calculation that in most instances the interest (I) earned or charged to the account has been rounded to two decimals. This will cause the calculation of t to be slightly imprecise. Therefore, when calculating t it is possible that decimals close to an integer (such as 128.998 days or 130.012 days) may show up. These decimals should be rounded to the nearest integer to correct for the rounded interest amount.

Part 3 Section 9.2D

- In determining when it is appropriate to round, it is important to recognize that if the money remains inside an account (deposit or loan), all of the decimals need to carry forward into the next calculation. For example, if a bank deposit of \$2000 earns 6% p.a. compounded monthly for 4 years, and then earns 7% p.a. compounded quarterly for three more years, then the money remained in the account the whole time. We can solve this in one step as follows:

$$FV = 2000.00(1.005)^{48}(1.0175)^{12} = \$3129.06$$

Or two steps as follows:

$$FV = 2000.00(1.005)^{48} = \$2540.978322$$

$$FV = 2540.978322(1.0175)^{12} = \$3129.06$$

Note that the first step is an interim calculation, for which we must carry forward all the decimals to the next step where the solution can then be rounded.

- If money is withdrawn/transferred from the account at any time, then only 2 decimals can be carried forward to any further steps (since a currency payout can only involve 2 decimals).

Section 9.4C

- In promissory notes, the FV solution in the first step must be rounded to 2 decimals before discounting as this is the amount of the debt that will be repaid on the maturity date.

Section 9.5B

- When calculating equivalent values for more than one payment, each payment is a separate transaction (one could make each payment separate from any other payment) and therefore any equivalent value is rounded to two decimals before summing multiple payments.

Section 10.1

1. When determining the n for non-annuity calculations (lump-sum amounts), generally the solution would not be rounded off since n can be fractional in nature (we can get 4.5632 quarters).
 - (a) However, when n is discussed, the n may be simplified to 2 decimals so that it is easier to communicate. For example, if $n = 5.998123$ years this would mean a term of slightly under 6 years. However, when discussed it may be spoken simply as a term of 6.00 years. Alternatively if $n = 17.559876$ months this would mean a little more than half way through the 17th month. However, when discussed it may be spoken as a term of approximately 17.56 months.
 - (b) An exception to this rule is when the n gets converted into days. As interest generally is not accrued more than daily, a fraction of a day is not possible. The fraction shows up most likely due to rounding in the numbers being utilized in the calculation. Since we do not know how these numbers were rounded, it is appropriate for our purposes to round n to the nearest integer.

Section 11.5A

1. When determining the n for annuity calculations, remember that n represents the number of payments. Therefore, n must be a whole number and should always be rounded upwards. Whether a partial or full payment is made, it is still a payment. For example, if $n = 21.34$ payments, this would indicate 21 full payments and a smaller last payment (which is still a payment). Therefore, 22 payments are required.
 - (a) In most cases, the payment (PMT) has been rounded to two decimals. This may cause insignificant decimals to show up in the calculations. As a result, an exception to this rule would be when n is extremely close to a whole number. This would mean that no significant digits show up in the first two decimals. For example, if $n = 23.001$, it can be reasonably concluded that n is 23 payments since the 0.001 is probably a result of the rounded payment.

Section 13.1E

1. When working with the n for an annuity due, n represents the number of payments and must be a whole number. Therefore, n will always round upward. However, it is important to distinguish whether the question is asking about the term of the annuity due or when the last payment of the annuity due occurs.
 - (a) If the term is being asked, n can be used to figure out the timeline. For example, a yearly apartment rental agreement would have $n = 12$ monthly payments, thus the term ends 12 months from now.
 - (b) If the last payment is being asked, $n - 1$ can be used to figure out the timeline. In the same example, the last rental payment would occur at the beginning of the 12th month. The last payment would be $12 - 1 = 11$ months from now.

Section 14.1

1. The payment must be rounded to the two decimal standard for currency.
2. When constructing an amortization schedule, it is important to recognize that all numbers in the schedule need to be rounded to two decimals (since it is currency). However, since the money remains in the account at all times, all decimals are in fact being carried forward throughout. As such, calculated numbers may sometimes be off by a penny due to the rounding of the payment or the interest.

Section 15.1

1. When determining the purchase price for a bond, it is important to carry all the decimals until the calculation is complete. When completing the calculation by formula, the present value of the bond's face value and interest payments along with any accrued interest must be calculated. For simplicity, the text shows each of these values rounded to two decimals and then summed to get the purchase price. Remember though that all decimals are being carried forward until the final answer.

Section 15.5

1. A sinking fund schedule has the same characteristics as an amortization schedule and may also experience a penny difference due to the rounding of the payment or the interest.

Section 16.1

1. When making choices between various alternatives, it is sufficient to calculate answers rounded to the nearest dollar. There are two rationales for this. First, in most cases future cash flows are not entirely certain (they are estimates) and therefore may be slightly inaccurate themselves. Second, as cents have little value, most decisions would not be based on cents difference; rather decisions would be based on dollars difference.

Section 16.2

1. In choosing whether to accept or reject a contract using the net present value method, remember that future cash flows are estimates. Therefore, when an NPV is calculated that is within \$500 of \$0, it can be said that the result does not provide a clear signal to accept or reject. Although the desired rate of return has barely been met (or not), this may be a result of the estimated cash flows. In this case, a closer examination of the estimates to determine their accuracy may be required before any decision could be made.

Section 16.3

1. Performance indexes are generally rounded to one decimal in percentage format.
2. This unknown rate of return (d) is generally rounded to 2 decimals in percentage format.
3. A rate of return is generally rounded to one decimal in percentage format.

For Daryl, Kirkland, and Kealeigh.

— **K.H.**

To my family for their support and to my two angels, Emma and Elina,
who have brought so much happiness into our lives.

— **A.R.H.**

PART

1

Mathematics Fundamentals and Business Applications

- 1**
Review of Arithmetic
- 2**
Review of Basic Algebra
- 3**
Ratio, Proportion,
and Percent
- 4**
Linear Systems

The first four chapters and Appendix I call upon students to activate prior knowledge learned in their earlier mathematics courses. As such, Part 1 is intended to provide an opportunity to review arithmetic and algebraic processes and to apply these skills in relevant business situations in fields such as accounting, finance, marketing, human resources, and management.

Chapter 1 covers the basics of arithmetic operations. In this chapter you will learn how to set up equivalent fractions, convert fractions and mixed numbers into decimals and percents, evaluate complex fractions, reduce fractions to lowest terms, and simplify expressions using the rules of the order of operations.

Calculating percents is introduced early because, from a practical standpoint, it is often necessary to convert a percent to a decimal when performing arithmetic calculations or using a calculator.

Applications involving payroll, commissions, GST, PST, HST, and property taxes call upon the use of these basic arithmetical operations and percentages. With respect to payroll, you will be able to calculate regular pay, overtime pay, and total pay. The text illustrates how to calculate straight commission, graduated commission (sliding scale), and salary plus commission as part of a complete discussion on gross earnings.

Chapter 2 reviews the basics of algebra, including simplifying algebraic expressions, evaluating algebraic expressions by substituting numbers into the variables, solving algebraic equations, and creating and solving word problems. Examples show how positive, negative, fractional, and zero exponents are defined. The study of terms involving positive, negative, and zero exponents serves as a prelude to the introduction of logarithms. Logarithms are useful in solving equations in which the unknown is an exponent. These expressions involving exponents appear in the study of compound interest developed later in the text.

Problems involving ratios, proportions, and percents abound in the field of business, and so in Chapter 3 you will learn how to use ratios to solve allocation and equivalence

problems. Exchange rate comparisons and the Consumer Price Index (CPI) examples in the chapter demonstrate practical applications of proportions. Discounts, interest rates, growth in earnings, and wages all use percentages.

Chapter 4 deals with linear equations and systems of two simultaneous linear equations in two variables. The algebraic elimination method is demonstrated for solving a system of two linear equations. The rectangular coordinate system is introduced, and an ordered pair is defined. You will learn how to set up a table of ordered pairs that can be used to graph a linear equation. The slope-intercept form of the linear equation is introduced, and special cases are discussed. You will learn how to develop a linear equation to describe a relationship between two variables and how to set up a system of linear equations to solve word problems involving two variables.

The **Universal Principle of Rounding** applies in Part 1 of the textbook. When performing a sequence of operations, never round any interim solution until the final answer is achieved. Apply rounding principles only to the final answer. Interim solutions should only be rounded where common practice would require rounding.

Note, however, that due to space limitations, the textbook shows only the first six decimals (rounded) of any number.

Some specific rounding guidelines for Part 1: Business problems throughout the textbook often involve money values so the rounding for final answers needs to be done to the cent; that is, to two decimal places. However, because larger sums of money usually are involved in currency exchanges, the two-decimal rule is insufficient. To produce a more accurate result, currency exchange rates used in Chapter 3 need to carry at least four decimals. Also, note that in Chapter 3, price per litre of gasoline is generally expressed to three decimal points (129.9 cents = \$1.299/L).

While different methods of rounding are used, for most business purposes the following procedure is suitable:

1. If the first digit in the group of decimal digits that is to be dropped is 5 or greater, the last digit retained is *increased* by 1.
2. If the first digit in the group of decimal digits that is to be dropped is 4 or less, the last digit retained is left *unchanged*.

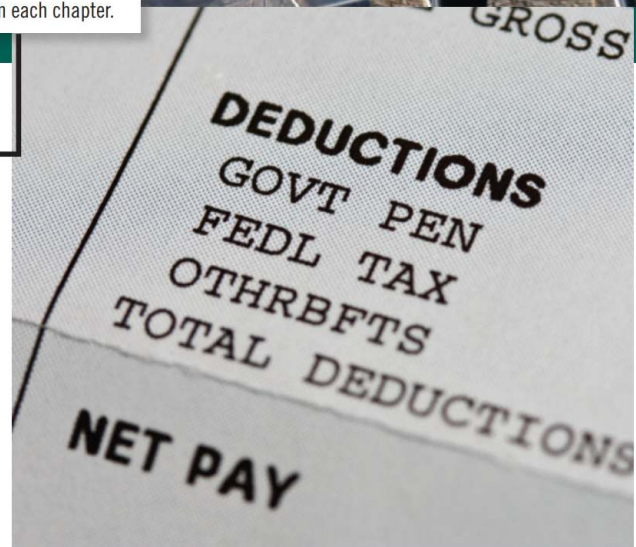
Review of Arithmetic

Objectives are a “roadmap” showing what will be covered and what is especially important in each chapter.

LEARNING OBJECTIVES

Upon completing this chapter, you will be able to do the following:

- 1 Simplify arithmetic expressions using the basic order of operations.
- 2 Determine equivalent fractions, and convert fractions to decimals, decimals to percents, and vice versa.
- 3 Through problem solving, compute simple arithmetic and weighted averages.
- 4 Determine gross earnings for employees remunerated by the payment of salaries, hourly wages, or commissions.
- 5 Through problem solving, compute GST, HST, PST, sales taxes, and property taxes.



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Each chapter opens with a description of a familiar situation to help you understand the practical applications of the material to follow.

Being able to perform arithmetic calculations is important in today's business environment. Arithmetic is used in planning, forecasting, purchasing, contracting, compensation, and many other aspects of business. Competence in problem solving, including calculation on averages, is essential. When you employ people in operating a business, you must determine the amounts to pay them in the form of salaries or wages, and you must deduct and pay payroll taxes such as Canada Pension Plan, Employment Insurance, and employee income taxes. You are responsible for paying your employees and submitting the tax amounts to the federal government. Operating a business also means that you must determine the amount of goods and services tax (GST) or harmonized sales tax (HST) to collect on almost everything you sell. The amount you must remit to the federal government, or the refund you are entitled to, is calculated on the basis of the GST or HST you pay when you make purchases of goods and services. By using arithmetic and problem-solving approaches in this chapter, you will be able to determine the amounts owed.

INTRODUCTION

The basics of fraction, decimal, and percent conversions are vital skills for dealing with situations you may face, not only as a small-business owner but also as a consumer and investor. Although calculators and computers are commonly used when performing arithmetic operations, to be able to solve more complex business problems modelled using algebra (and calculus), it is important to be skilled at the process of conversion between number forms, the rounding of answers, and the correct order of operations. (Appendix II at the back of the text provides basic operations for three common pre-programmed financial calculator models.)

1.1 BASICS OF ARITHMETIC

A. The basic order of operations

Boldfaced words are Key Terms that are explained here and defined in the Glossary section at the end of the chapter.

To ensure that arithmetic calculations are performed consistently, we must follow the **order of operations**.

If an arithmetic expression contains brackets, exponents, multiplication, division, addition, and subtraction, we use the following procedure:

1. Perform all operations *inside* a bracket first (the operations inside the bracket must be performed in proper order).
2. Perform exponents.
3. Perform multiplication and division.
4. Perform addition and subtraction.

The following “BEDMAS” rule might help you to more easily remember the order of operations:

B	E	D	M	A	S
Brackets	Exponents	Division	Multiplication	Addition	Subtraction

Numerous examples, often with worked-out Solutions, offer you easy-to-follow, step-by-step instructions.

EXAMPLE 1.1A

- (i) $(9 - 4) \times 2 = 5 \times 2 = 10$ ————— **work inside the bracket first**
- (ii) $9 - 4 \times 2 = 9 - 8 = 1$ ————— **do multiplication before subtraction**
- (iii) $18 \div 6 + 3 \times 2 = 3 + 6 = 9$ ————— **do division and multiplication before adding**
- (iv) $(13 + 5) \div 6 - 3 = 18 \div 6 - 3$ ————— **work inside the bracket first, then do**
 $= 3 - 3$ **division before subtraction**
 $= 0$
- (v) $18 \div (6 + 3) \times 2 = 18 \div 9 \times 2$ ————— **work inside the bracket first, then do**
 $= 2 \times 2$ **division and multiplication in**
 $= 4$ **order**
- (vi) $18 \div (3 \times 2) + 3 = 18 \div 6 + 3$ ————— **work inside the bracket first, then**
 $= 3 + 3$ **divide before adding**
 $= 6$
- (vii) $8(9 - 4) - 4(12 - 5) = 8(5) - 4(7)$ ————— **work inside the brackets first, then**
 $= 40 - 28$ **multiply before subtracting**
 $= 12$

(viii) $\frac{12 - 4}{6 - 2} = (12 - 4) \div (6 - 2)$ ————— the fraction line indicates brackets as well as division
 $= 8 \div 4$
 $= 2$

(ix) $128 \div (2 \times 4)^2 - 3 = 128 \div 8^2 - 3$ ————— work inside the bracket first, do the exponent, then divide before subtracting
 $= 128 \div 64 - 3$
 $= 2 - 3$
 $= -1$

(x) $128 \div (2 \times 4^2) - 3 = 128 \div (2 \times 16) - 3$ ————— start inside the bracket and do the exponent first, then multiply, then divide before subtracting
 $= 128 \div 32 - 3$
 $= 4 - 3$
 $= 1$

(xi) $12 - 5[8 - 2(9 - 3)] \div 2 = 12 - 5[8 - 2(6)] \div 2$ — work inside the square bracket first, then multiply before dividing by 2
 $= 12 - 5[8 - 12] \div 2$
 $= 12 - 5[-4] \div 2$
 $= 12 + 20 \div 2$
 $= 12 + 10$
 $= 22$

(xii) $8a + 3[10 - (2a + 1)] = 8a + 3[10 - (2a + 1)]$ — work inside the square bracket first, then multiply by 3 before combining like terms
 $= 8a + 3[10 - 2a - 1]$
 $= 8a + 3[9 - 2a]$
 $= 8a + 27 - 6a$
 $= 2a + 27$

EXERCISE 1.1

MyLab Math

Evaluate each of the following.

1. $12 + 6 \div 3$

3. $(7 + 4) \times 5 - 2$

5. $6(7 - 2) - 3(5 - 3)$

7. $4(8 - 5)^2 - 5(3 + 2^2)$

9. $250(1 + 0.08)^{10}$

11. $30 \times 600 - 2500 - 12 \times 600$

13. $15 - 7 + 6(2 + 3) \div 3$

15. $(1 - 0.7) - 4 \times 20 \div 5$

17. $7a - 6[4 - (3a + 6)]$

2. $(3 \times 8 - 6) \div 2$

4. $5 \times 3 + 2 \times 4$

6. $\frac{20 - 16}{15 + 5}$

8. $(3 \times 4 - 2)^2 + (2 - 2 \times 7^2)$

10. $(1 + 0.04)^4 - 1$

12. $1 - [(1 - 0.40)(1 - 0.25)(1 - 0.05)]$

14. $16 \div 2 \times 4 + 6(4 + 2)$

16. $50[(1 - 0.2)(1 - 0.175)(1 - 0.04)]$

18. $6a + 4b + 2(16 - 2a + b)$

Each section in the chapter ends with an exercise that allows you to review and apply what you've just learned. And you can find the solutions to the odd-numbered exercises at the back of the text.

For questions with a globe icon, visit MyLab Math to practise as often as you want. The guided solutions help you determine an answer step by step.

Reference Example 1.1A

References to examples direct you back to the chapter for help in answering the questions.